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ABSTRACTS OF TALKS

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TILTING THEORY AND THE FINITISTIC DIMENSION CONJECTURES

Lidia Angeleri Hügel

This is a report on joint work with Jan Trlifaj. We apply the theory of (infinitely generated) tilting modules in order to characterize when the little finitistic dimension of a right noetherian ring is finite. Moreover, we obtain a criterion for equality of the little and the big finitistic dimension that generalizes and refines the well-known criterion of Huisgen-Zimmermann and Smalo.

ON COVERS AND COVERING MORPHISMS

Robert El Bashir

We give a characterization of covering morphisms relative to a class of modules $\mathcal{F}$. We than consider flat covering morphisms of finitely generated modules over a commutative Noetherian ring. We distinguish different types of these due to their behavior with respect to the automorphism group of the flat cover.

STRONGLY FLAT COVERS AND ALMOST PERFECT DOMAINS

Silvana Bazzoni

An R-module M on a commutative domain R, is called *strongly flat* if $Ext^1_R(M, C) = 0$, for every R-module C satisfying $Ext^1_R(Q, C) = 0$ where Q denotes the field of quotients of R. We prove that the class of the strongly flat R-modules is a cover class if and only if it coincides with the class of flat R-modules. We also prove that a commutative domain R is such that the classes of flat R-modules and of strongly flat R-modules coincide if and only if for every non-zero ideal I of R the factor R/I

is a perfect ring. A domain R with this property is called almost perfect. We study the properties of this class of domains and we exhibit examples by means of three different constructions.

RELATIVE EXACT MODULES AND COVERS

Ladislav Bican

Recently, Rim and Teply found a necessary condition for the existence of σ-torsionfree covers with respect to a given hereditary torsion theory for the category R-mod. This condition uses the class of σ-exact modules, i.e the modules which are σ-torsionfree and every its σ-torsionfree homomorphic image is σ-injective. It is shown that the existence of σ-torsionfree covers implies the existence of σ-exact covers and some sufficient conditions for the converse are investigated.

ON AUSLANDER-GORENSTEIN RINGS AND EXCELLENT EXTENSIONS

John Clark

In this talk, we give a quick review of Auslander-Gorenstein rings, with some simple examples, followed by an introduction to excellent extensions with emphasis on property transfer.

τ-COMPLEMENTED MODULES VS τ-COMPLETELY DECOMPOSABLE MODULES

Septimiu Crivei

For a hereditary torsion theory τ, a module A is called τ-complemented if every submodule of A is τ-dense in a direct summand [Gol], [S]. Alongside of basic properties of certain τ-complemented modules, such as τ-complemented τ-injective modules and τ-completely decomposable modules, we will give a few decomposition theorems. Thus, for a ring having ACC both on τ-dense and τ-closed left ideals, a module is τ-complemented τ-injective if and only if it is τ-completely decomposable. We will also characterize the τ-injective hull of a τ-completely decomposable module and determine, for a commutative noetherian ring R that is not a domain, those torsion theories with the property that every τ-injective module is an essential extension of a τ-completely decomposable module.

References

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[MH] K. Masaike and T. Horigome,, *Direct sum of σ-injective modules*, Tsukuba J.Math. **4** (1980), 77-81.

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COMODULES FROM A TORSION THEORETIC POINT OF VIEW

J. Cuadra

It is well-known that for a coalgebra C its dual algebra C^* is a topological algebra with the weak-* topology. The category of right C-comodules $\mathcal{M}^C$ is isomorphic to the category of rational left C^*-modules $Rat(C^\star - Mod)$. This category may be considered as the hereditary pretorsion class associated to the linear topology $\mathcal{F}_C$ of all closed and cofinite left ideals of C^*. In this talk we study some examples of $Rat(C^\star - Mod)$ being a torsion class in $C^\star - Mod$, i.e, it is closed under extensions.

It was shown in [4] that if $\mathcal{F}_C$ has a basis of finitely generated ideals, then $Rat(C^\star - Mod)$ is a torsion class. We give some examples of coalgebras where the converse is true. This happens for almost connected coalgebras, cocommutative coalgebras, and right semiperfect coalgebras. Indeed, for a right semiperfect coalgebra C, $\mathcal{F}_C$ has a basis of principal ideals generated by an idempotent.

We also prove that a coalgebra C verifying that $Rat(C^\star - Mod)$ is a torsion class is a locally finite coalgebra. That is, for any finite dimensional subspaces U, V of C the wedge $U \wedge V$ is finite dimensional.

Finally we study when $Rat(C^\star - Mod)$ is a stable torsion class in $C^\star - Mod$. This is equivalent to $Rat(C^\star - Mod)$ having a finite number of simple modules and C^* being left noetherian.

References

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[2] J. Cuadra, $\mathcal{F}$-Noetherian Coalgebras. Preprint.

[3] B. I-Peng Lin, *Semiperfect Coalgebras*. J. Algebra **49** (1977), 357-373.

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FOXBY EQUIVALENCE AND COTORSION THEORIES

E. Enochs

Dualizing complexes (and modules) were defined by Grothendieck. Using these complexes Foxby introduced an equivalence between two classes of modules i.e. between the Auslander and Bass classes associated with the dualizing complex. We consider the question of the existence of a natural cotorsion theory related to these two classes when there does exist a dualizing module.

KRULL-SCHMIDT THEOREM: RECENT DEVELOPMENTS

Alberto Facchini

I will present some recent developments of the Krull-Schmidt Theorem. After a brief historical presentation, I will talk about the Krull-Schmidt Theorem for various algebraic structures. Then I will move on to consider the case of modules over associative rings. The case of serial modules will be presented (A module is uniserial if its lattice of submodules is linearly ordered by inclusion, and is serial if it is a direct sum of uniserial modules.) In this context the notions of monogeny classes and epigeny classes appear naturally. In particular, I will present a recent result by Diracca and myself about the uniqueness of monogeny classes for finite direct sums of uniform modules. Finally, I will consider the extension from local endomorphism rings to homogeneous semilocal endomorphism rings, that is, rings that are simple artinian modulo their Jacobson radical.

ON COSTAR MODULES AND GENERALIZATIONS OF MORITA DUALITY

Kent Fuller

Both quasi-progenerators and tilting modules over arbitrary rings induce equivalences between certain categories of modules that are generalizations of Morita equivalence. C. Menini and A. Orsatti introduced a generalization of these modules that have come to be called $*$-modules. Over the years, several authors have investigated a dual notion of quasi-progeneratos, (recently called quasi-duality modules); and cotilting modules, dual to tilting modules, have been a central topic of recent investigations in module theory. Both types of modules induce generalizations of Morita duality. Here we consider a class of modules that contains the quasi-duality modules and the cotilting modules. They are, in a sense, dual to $*$-modules so we call them *costar modules*.

RELATIVE HOMOLOGICAL ALGEBRA FOR THE PROPER CLASS ω_f

Grigory Garkusha

The proper class of f-monomorphisms is introduced. The corresponding relatively injective (respectively FP-injective and flat) modules are pure-injective fp-injective (respectively fp-injective and fp-flat) modules. The relative (co)homological functors Ext_f and Tor^f as well as the corresponding dimensions for rings and modules are studied. Moreover, it is shown that the relative derived category $D_f^+(R)$ and the derived category of the locally coherent Grothendieck category $(\text{mod–}R, \mathrm{Ab})/\lim S^R$ as well as K-theory $K'(R) = K(R\text{–mod})$ for the category of finitely presented left modules and K-theory of the abelian category of coherent objects of $(\text{mod–}R, \mathrm{Ab})/\lim S^R$ are naturally equivalent.

AUTOMORPHISM GROUPS OF GROUPS

Rüdiger Göbel

In 1962 Wolfgang Gaschütz (Journal of Algebra) showed that any finite p–group (not the cyclic group of order 2) has a proper outer automorphism. This was generalized by Zalesskii to infinite nilpotent groups. The class $\mathcal{C}$ of all locally finite p-groups is fairly close to the first mentioned classes of groups, thus Philip Hall raised the question about the existence of $\mathcal{C}$–groups G with trivial outer automorphism group $\operatorname{Out} G = \operatorname{Aut} G/\operatorname{Inn} G$. The existence of such a $\mathcal{C}$–group G with $\operatorname{Out} G = 1$ was shown in the universe $V = L$ by Simon Thomas, a result extended to ZFC and realizing all COUNTABLE groups as outer automorphism groups of $\mathcal{C}$–groups by Dugas, Göbel in 1990 (Journal of Algebra). In a recent paper Braun, Göbel removed countablity by application of an old result due to Kulikov. I will discuss this material.

TOPOLOGICAL TILTING MODULES

Enrico Gregorio

The tilting theory can be extended to Grothendieck categories; among these there are closed classes of modules, i.e., full subcategories of modules closed under submodules, quotients and direct sums. Some tilting equivalences in this case admit a representation through a topological tilting module; in this way we extend the classical theory to a larger setting, comprising that of comodules over coalgebras.

GORENSTEIN INJECTIVE MODULES AND Ext

Overtoun M. G. Jenda

Gorenstein injective modules can be characterized in terms of the extension functor with positive and negative indices. In this talk, we will show that over Iwanaga-Gorenstein rings, these modules can be characterized in terms of Ext with positive indices and countably presented modules of finite projective dimension.

A NOTE ON THE LATTICE OF HEREDITARY PRERADICALS

Josef Jirásko

Let $s \neq$ zer be a hereditary preradical. We give necessary and sufficient conditions for a hereditary prerasical r to be a unique co–atom in the lattice of all hereditary preradicals that are less or equal to s. We generalize some results of I. D. Bunu, Fundam. Prikl. Mat. **4** (1) (1998), 223–231.

ON WHEN SEMILOCAL COMMUTATIVE RINGS ARE FINITE

Tomáš Kepka

Various commutative semilocal rings satisfying some conditions of finiteness type are shown to be actually finite.

EXPLICIT DESCRIPTIONS OF THE INDECOMPOSABLE INJECTIVE MODULES OVER JATEGAONKAR'S RINGS

T. G. Kucera

In 1968–1969, A. V. Jategaonkar [J1], [J2] published his famous constructions of left but not right noetherian rings that provided counterexamples to several important conjectures of that era. These examples, and others like them, seemed to indicate that, in general, the task of completely understanding the structure of indecomposable injective modules over one-sided noetherian rings was hopeless. Indeed, in remarking about the difficulties associated with one-sided noetherian rings, Jategaonkar [[]p. 91]J3 says "Our decision to work with Noetherian rings rather than right Noetherian ones is often dictated by the exigencies of the situation under consideration. We note though that, after Jategaonkar (69) [that is, [J2] below], an attempt to stay with right Noetherian rings at all costs is generally regarded as futile." The family of examples developed in the paper cited are important and fascinating; they illustrate how strongly the left ideal structure of a ring can be disconnected from the right ideal structure of the ring. Nonetheless, they *do not* provide examples of indecomposable injectives that are difficult to understand or impossible to describe.

It is the purpose of the paper being reported on here not only to describe the indecomposable injectives over these badly behaved rings, but to convince the reader that the structure of the indecomposables is easily and naturally deduced from elementary facts about injective modules combined with the description of the rings themselves. In this talk I will outline how to deduce by natural methods, directly from the known description of these rings and their properties, explicit computational representations of the indecomposable injective left modules over Jategaonkar's rings. I use these explicit descriptions to answer some simple structural questions about the indecomposables.

NATURAL DUALITIES

F. Mantese and A. Tonolo

Let S be an arbitary associative ring and ${}_SW$ be a left S-module. Denote by R the ring $\mathrm{End}({}_SW)$ and by Δ both the contravariant functors $\mathrm{Hom}_S(-,W)$ and $\mathrm{Hom}_R(-,W)$. A module M is reflexive if the evaluation map $\delta_M : M \to \Delta^2 M$ is an isomorphism. Any direct summand of finite direct sums of copies of ${}_SW$ and R_R is reflexive. Increasing in a minimal way the classes of reflexive modules, a "cotilting condition" on finitely generated R-modules naturally arises.

COVERS AND ENVELOPES IN GROTHENDIECK CATEGORIES

Luis Oyonarte

The problem of the existence of covers by different classes of objects in abelian categories, giving special emphasis to the class of flat objects, has been studied by many different authors since the existence or flat covers in any module category over any arbitrary associative ring was conjectured by Edgar E. Enochs in 1981.

Recently, some developments of the study of cotorsion theories given by Paul C. EkIof and Jan Trlifaj have been used to solve the flat covers conjecture mentioned above. But this developments have been very useful not only for module categories, but for any other Grothendieck category. We present some results that reduce the problem of the existence of $\mathcal{F}$-covers and envelopes in a Grothendieck category with $\mathcal{A}$ enough projectives to the study of some "standard" properties of the class $\mathcal{F}$. Precisely we prove that:

1) when the class $\mathcal{F}$ is closed under direct sums, extensions and continuous well ordered unions, if the pair $(\mathcal{F}, \mathcal{F}^{\perp})$ is cogenerated by a set then evry object $M \in Ob(\mathcal{A})$ has a special $\mathcal{F}^{\perp}$-preenvelope.

2) If moreover $\mathcal{F}$ is closed under well ordered direct limits, then every $M \in Ob(\mathcal{A})$ has an $\mathcal{F}^{\perp}$-envelope. If furthermore the class $\mathcal{F}$ contains all projective objects of $\mathcal{A}$ then we have the same result for $\mathcal{F}$-precovers and $\mathcal{F}$-covers, that is, when 1) holds we always have $\mathcal{F}$-precovers, and when also 2) holds we always have $\mathcal{F}$-covers.

These results can finally be used to show the existence of flat covers in some particular interesting Grothendieck categories as the one of complexes of modules and the one of sheaves of modules over any topo-logical space. So we get that any complex of modules over any associative ring has a flat cover, and using that the category of presheaves over any topological space has enough projectives (the category of sheaves does not have enough projectives in general) we get that any sheaf of modules over any topological space has a flat cover.

LOCALLY FINITELY PRESENTED CATEGORIES OF MODULES

Mike Prest

A Grothendieck abelian category is *locally finitely presented* if its objects are determined by their elements.

Let $\mathcal{O}_X$ be a ringed space. The category of presheaves over $\mathcal{O}_X$ is locally finitely presented but the category of sheaves of $\mathcal{O}_X$–modules might not be. We give some conditions under which the latter category is locally finitely presented.

IDEAL LATTICES OF DIMENSION GROUPS

Pavel Růžička

A partially ordered abelian group G, with a positive cone G^+, is a *dimension group* provided

(1) $G = G^+ + (-G^+)$ (where $-G^+ = \{-g \mid g \in G\}$),
(2) if, for a positive integer n, $na \in G^+$, then $a \in G^+$, for every $a \in G$,
(3) for every $a_1, a_2, b_1, b_2 \in G$ with $a_1 + a_2 = b_1 + b_2$ there exist c_{ij}, $i, j = 1, 2$ with $a_i = c_{i1} + c_{i2}$ and $b_j = c_{1j} + c_{2j}$.

The semilattice $\mathrm{Id}^c(G)$ of finitely generated ideals of G is isomorphic to the maximal semilattice quotient of G^+ and, due to (3), it is distributive.

We prove that for every countable distributive semilattice S there exists a dimension group G such that $\mathrm{Id}^c(G) \simeq S$. On the other hand, we give an outline of an example of a distributive semilattice of size $\aleph_2$ which is not isomorphic to the semilattice of finitely generated ideals of any dimension group.

CLASSICAL COTORSION THEORIES OVER DOMAINS

Luigi Salce

We consider the three classes of modules over integral domains which generalize the class of cotorsion abelian groups: strongly cotorsion, cotorsion and weakly cotorsion modules. The connection via cotorsion theories with their Ext-orthogonal classes of torsionfree, flat and strongly flat modules is explained. A description of the strongly flat modules is provided, with special emphasis to the case of valuation domains. The existence of strongly flat covers is discussed, and the domains whose (finitely generated) modules admit these covers are characterized. The comparison with classical results by Bass on perfect and semi-perfect rings is considered.

ON LIE p-ALGEBRAS

Mirela Stefanescu and Camelia Ciobanu

Some properties of Lie p-algebras, concerning their representations and cohomology are studied in the paper.

A Lie p–algebra is a Lie algebra $\mathcal{L}$ over a field k of characteristic $p > 0$, endowed with a p–mapping $x \to x^{[p]}$, $x \in \mathcal{L}$, satisfying the properties:

(i) $(\alpha x)^{[p]} = \alpha^p x^{[p]}$,

(ii) $(x+y)^{[p]} = x^{[p]} + y^{[p]} + \sum_{i=1}^{p-1} s_i(x,y)$, where $is_i(x,y)$ is the coefficient of λ in the decomposition of $x(\mathrm{ad}(\lambda x + y))^{p-1}$,

(iii) $\left[x, y^{[p]}\right] = x(\mathrm{ad})\, y)^p$, for all $\alpha \in k$ and $x, y \in \mathcal{L}$.

The Frattini p-subalgebra $\mathcal{F}_p(\mathcal{L}) = \bigcap \{\, \mathcal{A} \mid \mathcal{A}$ is a maximal p–subalgebra of $\mathcal{L}\}$ and the Frattini p–ideal $\Phi_p(\mathcal{L})$, the greatest p–ideal in $\mathcal{F}_p(\mathcal{L})$, are used to characterize the solvable and nilpotent Lie p–algebras with some other properties.

DECISION PROBLEMS FOR TORSIONFREE MODULES OVER GROUP RINGS

Carlo Toffalori

This is a joint work with Saverio Cittadini. Let R be a (countable) Dedekind domain, G be a finite group. We compare the representation type of RG-lattices and the decidability of the class of arbitrary R-torsionfree RG-modules.

LOCAL SPLITTERS FOR BOUNDED COTORSION THEORIES

Jan Trlifaj

Enochs' proof of the Flat Cover Conjecture as well as recent applications of tilting theory to Finitistic Dimension Conjectures (Angeleri-Trlifaj) are based on the fact that the relevant cotorsion theories are bounded, and hence provide for special approximations of modules. In fact, Gödel's Axiom of Constructibility implies that any singly generated cotorsion theory over any hereditary ring is bounded (Eklof-Trlifaj), and so is the W^2-cotorsion theory over any IC-domain (Bazzoni-Salce). The latter results, however, are not provable in ZFC:

Assume Shelah's Uniformization Principle. Let $\mathcal{C}$ be a bounded cotorsion theory and K be a local splitter for $\mathcal{C}$. Then $\mathcal{C}$ is not generated by K. Taking $K = R$ and $K = Q/R$, we get respectively (1) the existence of Whitehead modules of projective dimension 1 over any non-right perfect ring R, and (2) the existence of W^2-modules of projective dimension 2 over any Matlis valuation domain R of global dimension 2. Restricting to countable Dedekind domains in (1), and IC-domains in (2), we get in either case that the cotorsion theory generated by K is not bounded.

ω_1–GENERATED UNISERIAL MODULES OVER CHAIN RINGS

Jan Žemlička

The purpose of this talk is to provide a criterion of an occurence of uncountably generated uniserial modules over chain rings. It suffices to investigate two extreme cases, nearly simple rings, i.e. rings containing only three two–sided ideals, and rings with "many" two–sided ideals. We prove that there exist ω_1–generated uniserial modules over every non–artinian nearly simple chain ring and over rings containing uncountable strictly increasing (resp. decreasing) chain of right (resp. two–sided) ideals. As a consequence, right steady serial rings are described.