Some Trends in Algebra '09

Prague, September 8 – 11, 2009

A B S T R A C T S O F T A L K S

Bounded movements permutation groups with maximum orbits

Mehdi Alaeiyan

Let G be a permutation group on a set Ω with no fixed points in Ω and let m be a positive integer. If for a subset Γ of Ω the size $|\Gamma^g - \Gamma|$ is bounded, for $g \in G$, we define the movement of Γ as

$$move(\Gamma) := sup_{g\in G}|\Gamma^g - \Gamma|.$$

If $move(\Gamma) \leq m$ for all $\Gamma \subseteq \Omega$, then G is said to have bounded movement and the movement of G is defined as the

$$m := move(G) := \sup_{\Gamma} |\Gamma^g - \Gamma|.$$

This notion was introduced in [2,3]. By [3, Theorem 1], if G has bounded movement m, then Ω is finite. Moreover both the number of G-orbits in Ω and the length of each G-orbit are bounded above by linear functions of m. In particular each G-orbit has length at most $3m$, $t \leq 2m - 1$ and $n := |\Omega| \leq 3m + t - 1 \leq 5m - 2$, where t is the number of G-orbits on Ω. It was shown that $n = 5m - 2$ if and only if $n = 3$ and G is transitive. But this bound was refined further and it was shown that $n \leq (9m - 3)/2$. Moreover, if $n = (9m - 3)/2$ then either $n = 3$ and $G = S_3$ or G is an elementary abelian 3-group and all its orbits have length 3.

These results can be refined further, if we restrict the structure of an abstract group G. Suppose now that G is not a 2-group, and let p be the least odd prime divisor of $|G|$. Assume that $p \geq 5$. In this case, the bound $n \leq (9m - 3)/2$ was obtained in [3, , Lemma 2.2], which was improved to the bound $n \leq 4m - p + 3$.But this bound was refined further in [1] and it was proved that $n \leq 4m - p$. Moreover, if $n = 4m - p$ then G is isomorphic to either $(Z_p.Z_{2^a}) \times Z_2^d$ or $(Z_p.Z_{2^a}) \times Z_2^{d-1}$ with $m = (p-1)/2 + 2^{d-1}$ for some non-negative integer a with $2^a|p - 1$. In particular in the transitive case $|\Omega| \leq 2mp/(p - 1)$. Also in the case of equality the classification has been done in [2].

Example Let d be a positive integer, p a prime, $G := Z_p^d$, $t := (p^d - 1)/(p - 1)$, and let $H_1, \ldots, H_t$ be all subgroups of index p in G. Define Ω_i to be the right coset space $\{H_i g | g \in G\}$ of H_i and $\Omega := \Omega_1 \cup \cdots \cup \Omega_t$. Consider G as a permutation group on Ω by the right multiplication, that is $x \in G$ is identified with the composite of permutation $H_i g \longmapsto H_i gx$ $(i = 1, \ldots, t)$ on Ω_i for $i = 1, \ldots, t$. If $g \in G - \{1\}$, then g lies in $(p^{d-1} - 1)/(p - 1)$ groups H_i and therefore acts on Ω as a permutation with $p(p^{d-1} - 1)/(p - 1)$ fixed points and p^{d-1} orbits of length p. Taking every second point from each of these p-cycles to form a set Γ we see that move$(g) = m \geq p^{d-1}(p - 1)/2$ if p is odd or 2^{d-1} if $p = 2$, and it is not hard to prove that in fact move

$(g) = m = p^{d-1}(p-1)/2$ if p is odd or 2^{d-1} if $p = 2$. Since g is non-trivial, all non-identity elements of G have the same movement m.

These families of groups are the examples that satisfying in the maximum value of orbits.

Also, as a main result of this paper, we will show that if $n = |\Omega| = 3m + t - 1 = \lfloor (9m-3)/2 \rfloor$ with $t \geq 2$, then G is one of the following:

(i) G is an elementary abelian 3-group and all its orbits have length 3.

(ii) G is the semidirect product of $\mathbb{Z}_2^2$ by $\mathbb{Z}_3$ with normal subgroup $\mathbb{Z}_2^2$.

References

[1] M. Alaeiyan and S. Yoshiara, Permutation groups of minimal movement, Arch. Math. **85** (2005), 211-226.

[2] A. Hassani, M. Khayaty (Alaeiyan), E. I. Khukhro and C. E. Praeger, Transitive permutation groups with bounded movement having maximal degree, *J. Algebra* **214** (1999), 317–337.

[3] C. E. Praeger, On permutation group with bounded movement, *J. Algebra* **144** (1991), 436–442.

Envelopes and covers

Silvana Bazzoni

The approximation theory for modules over arbitrary associative rings was introduced by Enochs in the eighties.

A relevant problem in this context is to decide if a module admits a minimal approximation with respect to a class $\mathcal{C}$ of modules, or in Enoch's terminology, if a module has a $\mathcal{C}$-cover or a $\mathcal{C}$-envelope. By Enochs and Xu's results, the closure under direct limits of the class $\mathcal{C}$ provides a sufficient condition for the existence of $\mathcal{C}$-covers and it is still an open problem to decide if this is also a necessary condition.

In [1] is asked to characterize the tilting cotorsion pairs $(\mathcal{A}, \mathcal{B})$ for which $\mathcal{B}$-envelopes always exist. We solve the problem for the tilting cotorsion pair $(\mathcal{A}, \mathcal{B})$ where $\mathcal{B}$ is the class of divisible modules over a commutative domain, so that $\mathcal{A}$ is the class $\mathcal{P}_1$ of modules of projective dimension at most 1. We show that every module has a divisible envelope if and only if $\mathcal{P}_1$ is closed under direct limits and this condition is known to characterize almost perfect domains, i.e. domains such that all their proper quotients are perfect rings.

Moreover, we show that $\mathcal{P}_1$-covers always exist if and only if the domain is almost perfect.

References

[1] R. Göbel, J, Trlifaj, **Approximations and endomorphism algebras of modules**, De Grutyter Expositions in Mathematics **41**, de Gruyter, Berlin 2006.

Lattice properties of torsion theories

Ladislav Bican

The lecture is devoted to some investigations leading to the facts that the sets of all hereditary torsion theories which are noetherian, strongly noetherian or of finite type, respectively, form a sublattice of the lattice of all hereditary torsion theories for the category R-mod.

Modules determined by their annihilator classes

Simion Breaz

This is a report on joint work with Jan Trlifaj. We present a classification of those finite length modules X over a ring A which are isomorphic to every module Y of the same length such that $\mathrm{Ker}(\mathrm{Hom}_A(-,X)) = \mathrm{Ker}(\mathrm{Hom}_A(-,Y))$, i.e. X is determined by its length and the torsion pair cogenerated by X. We also prove the dual result using the torsion pair generated by X. For A right hereditary, we prove an analogous classification using the cotorsion pair generated by X, but show that the dual result is not provable in ZFC.

Monads, comonads and approximations

Septimiu Crivei

We discuss some concepts of the theory of monads and comonads, and we explore their behavior with respect to approximations.

Orthogonal classes with respect to selforthogonal modules

Gabriella D'Este

We compare more or less big classes of modules contained in the orthogonal class associated to selforthogonal modules

Rings with few maximal right ideals - A two-dimensional Krull-Schmidt-Azumaya Theorem

Alberto Facchini

I will present three recent papers written with P. Příhoda, A. Amini and B. Amini. We will first consider the behaviour, as far as direct-sum decompositions of direct sums of finitely many modules are concerned, of modules whose endomorphism rings have finitely many maximal right ideals, all of them two-sided ideals. The key concept turns out to be a particular kind of ideals in the category Mod-R. We will give several examples. In particular, we will consider the case of RD-projective modules over semiperfect rings. Then we will pass to consider the case in which the endomorphism rings have at most two maximal right ideals (in this case, the at most two maximal right ideals are always necessarily two-sided). The tool that describes the situation is a graph with a very simple structure. Its connected components are either complete graphs or complete bipartite graphs. We will conclude with a form of the Krull-Schmidt-Azumaya Theorem that generalizes the classical case of modules whose endomorphism ring is local, that is, with one maximal right ideal, to the case in which the endomorphism rings of the modules have at most two maximal right ideals.

Syntax-semantics duality: retrieving a theory from its category of models

Henrik Forssell

Solution of a 50 year old problem by Laszlo Fuchs: The construction of generalized E-rings

Rüdiger Göbel

Applications of cotorsion pairs to Model Categories

Pedro Antonio Guil Asensio

We will present several examples of constructions of model structures by means of cotorsion pairs. We particular, we will center our attention into three specially interesting applications: the cohomology of quasi-coherent sheaves over quasi-compact and semiseparated schemes; the so-called excision problem in K-theory for non-unital algebras; and the finitistic dimension conjecture for Artin Algebras.

Almost free modules and Mittag–Leffler conditions

Dolors Herbera

Drinfeld recently suggested to replace projective modules by the flat Mittag–Leffler ones in the definition of an infinite dimensional vector bundle on a scheme X. Two questions arise: (1) What is the structure of the class $\mathcal{D}$ of all flat Mittag–Leffler modules over a general ring? (2) Can $\mathcal{D}$ be used to build a Quillen model category structure on the category of chain complexes of quasi–coherent sheaves on X?

We answer (1) by showing that a module M is flat Mittag–Leffler, if and only if M is $\aleph_1$–projective in the sense of Shelah, Eklof, and Mekler. We use this to characterize the rings such that $\mathcal{D}$ is closed under products, and relate the classes of all Mittag–Leffler, strict Mittag–Leffler, and separable modules. Then we prove that the class $\mathcal{D}$ is not deconstructible for any non-right perfect ring, which yields a negative answer to (2).

This talk is based on a joint work with Jan Trlifaj.

The Ziegler spectrum of a cotilting class

Ivo Herzog

Let $\mathcal{C} \subseteq R-Mod$ be a cotilting class of left R-modules. By Bazzoni's Theorem, any cotilting module C for $\mathcal{C}$ is pure-injective. The functor $-\otimes_R C$ is therefore an injective object in the category $(mod-R, Ab)$ of additive functors $F : mod-R \to Ab$. We will consider the localization of $(mod-R, Ab)$ with respect to the hereditary torsion theory cogenerated by $- \otimes_R C$, and give a torsion-theoretic characterization of its simple objects.

Auslander-Reiten theory for modules of finite complexity over selfinjective algebras

Otto Kerner

If Λ is a selfinjective algebra over an algebraically closed field K, and M is a finite dimensional Λ-module with minimal projective resolution

$$\cdots \to P_n \to \cdots \to P_0 \to M,$$

then M has complexity at most d, if $\dim P_n \leq cn^{d-1}$ for all n and some positive number c. In a joint paper with Dan Zacharia we describe the shapes of those Auslander-Reiten components $\mathcal{C}$ in the Auslander-Reiten quiver $\Gamma(\Lambda)$ of Λ, containing a nonprojective module of finite complexity.

A Quillen model structure for Gray-categories

Stephen Lack

There is a well-known Quillen model structure on the category Cat of categories and functors for which the weak equivalences are the equivalences of categories. This restricts to a model structure on the full subcategory Gpd of Cat consisting of the groupoids, and this provides a model for (not necessarily connected) homotopy 1-types.

I will describe how this "$n = 1$" case (1-categories, 1-groupoids, 1-types) can be extended to the cases of $n = 2$ and $n = 3$. In each of these cases there is a model structure on the category of all n-dimensional categories. Once again, there is a restricted model structure on the full subcategory consisting of those n-dimensional categories for which all arrows at all dimensions are invertible; and this provides a model for n-types. In the case $n = 3$, the n-dimensional categories considered are the Gray-categories of the title.

I will build up from $n = 1$ to $n = 2$ and $n = 3$ in an inductive sort of way, although I do not know how to continue the induction to deal with higher n.

Brown representability via projective classes

George Ciprian Modoi

Extending an idea of Neeman we prove that a triangulated category with coproducts satisfies Brown representability theorem, provided that any of its objects is isomorphic to a homotopy colimit of a tower of objects, each of them having finite projective dimension with respect a suitable projective class.

Tilting modules over 1 and 2-Gorenstein Rings

David Pospíšil

We give a characterization (up to equivalence) of all tilting modules over 1-Gorenstein commutative rings using associated prime ideals. And we give a partial result in 2-Gorenstein case: characterization of all 2-tilting modules which are not 1-tilting.

Factor categories and infinite direct sums

Pavel Příhoda

In our joint work with A. Facchini we proved that direct summands of serial modules are described up to isomorphism by a set of cardinal invariants. This can be done adapting the standard technique of Harada and Sai. In this talk I will discuss the potential of this method for direct sums of modules with semilocal endomorphism rings.

Generalized purity, definability and Brown representability

Jiří Rosický

All three concepts (purity, definability and Brown representability) will be considered for uncountable cardinals.

Conical refinement monoids and $V(R)$ of regular rings

Pavel Růžička

We will study a question of a realization of countable conical monoids as $V(R)$ of regular F-algebras. We present several auxiliary results, including the existence of a Banaschewski function on countable regular rings, the existence of V-measures on countable conical refinement monoids and the dependence on the cardinality of the field F.

Adjoint algebraic entropy

Luigi Salce

I will present the new notion of adjoint algebraic entropy for endomorphisms of Abelian groups, whose behaviour is for certain aspects dual to that of the algebraic entropy. The connection with the Pontryagin duality will be investigated and its applications to the Addition Theorem and to the computation of the adjoit entropy of the shifts. Finally, the dichotomy theorem will be proved.

Cotorsion pairs in exact categories and construction of triangulated adjoint functors

JAN ŠŤOVÍČEK

(joint with Manuel Saorin)

When comparing the theory for cotorsion pairs in modules categories on one hand, and the way one works with t-structures and Bousfield localizations in triangulated categories on the other hand, one observes many formal similarities.

Inspired by recent work of several authors, the aim of this talk is to make this connection precise for triangulated categories coming from algebra. The crucial notions in this context, explained in the course of the talk, are exact categories and deconstructible classes. At the same time, the goal is to present easily applicable statements following a few simple guiding principles:

1. fundamental results about cotorsion pairs work unchanged for a diverse variety of exact categories;
2. it is often convenient to consider different exact structures on the same category;
3. the deconstructibility property is kept under certain natural constructions.

Rings over which every module is retractable

JAN ŽEMLIČKA

A module M is called retractable if there exists a non-zero homomorphism of M into every non-zero submodule $N \subseteq M$. We present several sufficient ring-theoretical conditions of rings over which all modules are retractable. A characterization of this property is given for particular classes of rings.