Torsion in tensor products and vanishing of Tor

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Let M and N be non-zero finitely generated modules over a local integral domain R. Typically, even when M and N are well behaved, for example, torsion-free or maximal Cohen-Macaulay (MCM), the tensor product $M \otimes_R N$ is not so well behaved. For example, Craig Huneke and I showed, about twenty years ago, that when R is a hypersurface $M \otimes_R N$ is *never* MCM unless either M or N is free. When studying depth properties of $M \otimes_R N$, such as whether or not the tensor product is torsion-free, one is led naturally to questions concerning vanishing of Tor. For example, if R is a complete intersection and $\operatorname{Tor}_i^R(M, N) = 0$ for all $i > 0$, one has the "Depth Formula":

$$\operatorname{depth} M + \operatorname{depth} N = \operatorname{depth} R + \operatorname{depth} M \otimes_R N$$

Thus one seeks conditions that force the vanishing of all higher Tors.

I will survey results on vanishing of Tor and related questions on the existence of torsion in tensor products. I will discuss, in particular, the following

Conjecture: If $M \otimes_R \operatorname{Hom}_R(M, R)$ is maximal Cohen-Macaulay, then M is free.

The conjecture is open even for one-dimensional complete intersections of codimension two. For a one-dimensional Gorenstein domain, the torsion submodule of $M \otimes_R \operatorname{Hom}_R(M, R)$ is the Matlis dual of $\operatorname{Ext}_R^1(M, M)$. An affirmative answer to this conjecture for one-dimensional Gorenstein domains would imply an affirmative answer to the celebrated Auslander-Reiten conjecture for Gorenstein domains of *any* dimension. A substantial part of the talk will deal with two on-going joint projects—one with O. Celikbas, S. Iyengar, and G. Piepmeyer, the other with Huneke and Iyengar.