ON SEPARATING SETS OF WORDS IV

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Abstract. Further properties of transitive closures of special replacement relations in free monoids are studied.

1. Introduction

This article is an immediate continuation of [1], [2] and [3]. References like I.3.3 (II.3.3, III.3.3, resp.) lead to the corresponding section and result of [1] ([2], [3], resp.) and all definitions and preliminaries are taken from the same source.

2. Complementary sequences

Troughout this note, let $Z \subseteq A^+$ be a strongly separating set of words and let $\psi : Z \to A^*$ be a mapping with $\psi(z) \neq z$ for every $z \in Z$. Notice that then the corresponding replacement relation ρ ($= \rho_{Z,\psi}$) is irreflexive.

Two sequences $p_0, p_1, \dots, p_m$ and $q_0, q_1, \dots, q_m$, $m \geq 1$, of words will be called *$((Z,\psi)-$ or ρ-) complementary* if, for every $0 \leq i < m$, either $(p_i, p_{i+1}) \in \rho$ and $q_i = q_{i+1}$ or $p_i = p_{i+1}$ and $(q_i, q_{i+1}) \in \rho$. Notice that due to the irreflexivity of ρ, just one of the two cases holds.

Lemma 2.1. *Let $p_0, p_1, \dots, p_m$ and $q_0, q_1, \dots, q_m$ be complementary sequences. Then:*

(i) *Both the sequences are λ-sequences.*
(ii) $(p_0, p_m) \in \xi$ *and* $(q_0, q_m) \in \xi$.
(iii) *If $(p_0, p_m) \notin \tau$ ($(q_0, q_m) \notin \tau$, resp.), then $p_0 = p_1 = \dots = p_m$ ($q_0 = q_1 = \dots = q_m$, resp.), $q_0, q_1, \dots, q_m$ ($p_0, p_1, \dots, p_m$, resp.) is a ρ-sequence and $(q_0, q_m) \in \tau$ ($(p_0, p_m) \in \tau$, resp.).*
(iv) *Either $(p_0, p_m) \in \tau$ or $(q_0, q_m) \in \tau$.*

Proof. Easy. □

Let $w_0, w_1, \dots, w_m$ be a ρ-sequence and let $z \in Z$. Furthermore, let $\alpha_i = (p_i, z_i, q_i) \in \mathrm{Tr}(w_i)$ (so that $w_i = p_i z q_i$) for all $i = 0, 1, \dots, m$. We will say that the ρ-sequence is *$(z, \alpha_0, \dots, \alpha_m)$-fluent* if the sequences $p_0, p_1, \dots, p_m$ and $q_0, q_1, \dots, q_m$ are complementary.

The work is a part of the research project MSM0021620839 financed by MŠMT and the second author was supported by the Grant Agency of Czech Republic, No. 201/09/0296.

Lemma 2.2. *Let $(w_0, w_1) \in \rho$ and $\alpha = (p_0, z, q_0) \in \mathrm{Tr}(w_0)$. Then $w_0 = p_0 z q_0$ and at least one of the following two cases holds:*

(1) *$w_0 \xrightarrow{\alpha} w_1$, $(w_0, w_1) \in \rho_z$ and $w_1 = p_0\psi(z)q_0$;*
(2) *$w_1 = p_1 z q_1$ and the sequences p_0, p_1 and q_0, q_1 are complementary (and hence the sequence w_0, w_1 is (z, α, β)-fluent, $\beta = (p_1, z, q_1)$).*

Proof. Assume that (1) is not true. Then there is $\gamma = (r, z_1, s) \in \mathrm{Tr}(w_0)$ such that $\gamma \neq \alpha$ and $w_1 = r\psi(z_1)s$. We have $p_0 z q_0 = w_0 = r z_1 s$, where $p_0 \neq r$ and $q_0 \neq s$. Consequently, $|p_0| \neq |r|$ and $|q_0| \neq |s|$.

First, assume that $|p_0| < |r|$. Then $r = p_0 r_1$, $r_1 \neq \varepsilon$, $z q_0 = r_1 z_1 s$, $|q_0| > |s|$, $q_0 = s_1 s$, $s_1 \neq \varepsilon$ and $z s_1 = r_1 z_1$. From this, $r_1 = zt$ and $s_1 = t z_1$ and we get $w_0 = p_0 z q_0 = p_0 r_1 z_1 s = p_0 z t z_1 s$, $q_0 = t z_1 s$, $r = p_0 z t$, $w_1 = r\psi(z_1)s = p_0 z t \psi(z_1) s = p_1 z q_1$, where $p_0 = p_1$, $q_1 = t\psi(z_1)s$ and $(q_0, q_1) \in \rho$.

Next assume that $|r| < |p_0|$. Then $p_0 = r r_1$, $r_1 \neq \varepsilon$, $r_1 z q_0 = z_1 s$, $|r_1| \geq |z_1|$, $r_1 = z_1 t$, $s = t z q_0$, $p_0 = r z_1 t$. Now, $w_0 = r z_1 t z q_0$, $w_1 = r\psi(z_1)s = r\psi(z_1) t z q_0 = p_1 z q_1$, where $q_0 = q_1$, $p_1 = r\psi(z_1)t$ and $(p_0, p_1) \in \rho$.

The lemma follows easily from I.6.4 as well. □

Lemma 2.3. *Let $w_0, w_1, \dots, w_m$ be a ρ-sequence and let $\alpha_0 = (p_0, z, q_0) \in \mathrm{Tr}(w_0)$ (so that $w_0 = p_0 z q_0$). Then at least one of the following two cases holds:*

(1) *$w_0 \xrightarrow{\alpha_0} w_1$, $(w_0, w_1) \in \rho_z$ and $w_1 = p_0\psi(z)q_0$;*
(2) *There are $1 \leq n \leq m$ and $\alpha_i = (p_i, z, q_i) \in \mathrm{Tr}(w_i)$ (so that $w_i = p_i z q_i$), $0 \leq i \leq n$, such that the sequence $w_0, w_1, \dots, w_n$ is $(z, \alpha_0, \alpha_1, \dots, \alpha_n)$-fluent and either $n = m$ or $n < m$ and $w_{n+1} = p_n\psi(z)q_n$ (so that $(w_n, w_{n+1}) \in \rho_z$ and $w_n \xrightarrow{\alpha_n} w_{n+1}$).*

Proof. Assume that (1) is not true and proceed by induction on m. If $m = 1$ then 2.2 applies. If $m \geq 2$, we consider the sequence $w_1, w_2, \dots, w_m$. □

Remark 2.4. Consider the situation from 2.3 (2) and assume that $n < m$. Put $v_0 = p_0 z q_0 = w_0$, $v_1 = p_0\psi(z)q_0$, $v_2 = p_1\psi(z)q_1$, $\dots$, $v_n = p_{n-1}\psi(z)q_{n-1}$ and $v_{n+1} = p_n\psi(z)q_n = w_{n+1}$. Clearly, $w_0 = v_0, v_1, \dots, v_n, v_{n+1} = w_{n+1}$ is a ρ-sequence, $v_0 = w_0 \xrightarrow{\alpha_0} v_1$, $w_1 \xrightarrow{\alpha_1} v_2$, $\dots$, $w_n \xrightarrow{\alpha_n} v_{n+1} = w_{n+1}$ and $w_0 = v_0, v_1, \dots, v_n, v_{n+1} = w_{n+1}, w_{n+2}, \dots, w_m$ is a ρ-sequence. In particular, $(p_0\psi(z)q_0, w_m) = (v_1, w_m) \in \tau$.

3. Auxiliary results (a)

Lemma 3.1. *Let $z_1, z_2 \in Z$ and $r \in A^*$. Then $z_1 r\psi(z_2) = \psi(z_1) r z_2$ iff at least (and then just) one of the following two cases holds:*

(1) *There are $s, t \in A^+$ such that $sr = rt$ (see I.3.5) and $z_1 = \psi(z_1)s$, $z_2 = t\psi(z_2)$;*

(2) *There are $s, t \in A^+$ such that $sr = rt$ and $\psi(z_1) = z_1 s$, $\psi(z_2) = tz_2$.*

Proof. Easy. □

Corollary 3.2. *The following two conditions are equivalent:*

(i) $z_1 r\psi(z_2) \neq \psi(z_1) r z_2$ *for all* $z_1, z_2 \in Z$ *and* $r \in A^*$.

(ii) $sr \neq rt$ *for every* $r \in A^*$ *whenever* $s, t \in A^+$ *and* $z_1, z_2 \in Z$ *are such that either* $z_1 = \psi(z_1)s$ *and* $z_2 = t\psi(z_2)$ *or* $\psi(z_1) = z_1 s$ *and* $\psi(z_2) = tz_2$ *(the latter case does not take place when* ψ *is strictly length decreasing).*

Lemma 3.3. *Let* $(w_0, w_1) \in \rho$ *and* $\alpha = (p_0, z, q_0) \in \mathrm{Tr}(w_0)$ *(see 2.2). If the equivalent conditions of 3.2 are satisfied, then just one of the cases 2.2 (1), (2) holds.*

Proof. If both 2.2 (1), (2) are true, then $p_0\psi(z)q_0 = w_1 = p_1 z q_1$ and either $p_0 = p_1$ and $(q_0, q_1) \in \rho$ or $(p_0, p_1) \in \rho$ and $q_0 = q_1$. Assume the first case, the other one being similar. Then $\psi(z)q_0 = zq_1$, $q_0 = rz_1 s$, $q_1 = r\psi(z_1)s$ and $\psi(z)rz_1 = zr\psi(z_1)$. The rest is clear from 3.2. □

Remark 3.4. Assume that the equivalent conditions of 3.2 are satisfied and let $(w_0, w_1) \in \rho$. Then it follows from 3.3 that $w_0 \overset{\alpha_0}{\to} w_1$ for a uniquely determined instance $\alpha \in \mathrm{Tr}(w_0)$.

Remark 3.5. Assume that the equivalent conditions of 3.2 are satisfied and consider the situation from 2.3. Then just one of the cases 2.3 (1), (2) holds. Furthermore, if 2.3 (2) is true, then the number n and the instances $\alpha_0, \alpha_1, \dots, \alpha_n$ are determined uniquely.

4. Auxiliary results (b)

In this section, let $z_1, z_2 \in Z$, $z_1 \neq z_2$, $r_1, r_2, s_1, s_2 \in A^*$, $t_1 = r_1 z_1 s_1$ and $t_2 = r_2 z_2 s_2$.

Lemma 4.1. $t_1 \neq t_2$ *in each of the following six cases:*

(1) $r_1 = r_2$;
(2) $s_1 = s_2$;
(3) r_1, s_1 *are reduced;*
(4) r_2, s_2 *are reduced;*
(5) r_1, r_2 *are reduced;*
(6) s_1, s_2 *are reduced.*

Proof. Easy to see (use I.6.4). □

Lemma 4.2. *Assume that the mapping* ψ *is length decreasing. Then* $(t_1, t_2) \notin \tau$ *in each of the following three cases:*

(1) $|r_1| + |s_1| \leq |r_2| + |s_2|$, $|z_1| \leq |z_2|$ *and at least one of these inequalities is sharp (equivalently,* $|t_1| < |t_2|$*);*

(2) *r_1, s_1 are reduced, $|r_1| + |s_1| \le |r_2| + |s_2|$, $|\psi(z_1)| \le |z_2|$ and at least one of these inequalities is sharp;*
(3) *r_1, s_1 are reduced and $|\psi(z_1)| < |z_1|$.*

Proof. Easy (if r_1, s_1 are reduced and $(t_1, t_2) \in \tau$, then $(r_1\psi(z_1)s_1, t_2) \in \xi$). □

Corollary 4.3. *Assume that the mapping ψ is strictly length decreasing and the words r_1, r_2, s_1, s_2 are reduced. Then $(t_1, t_2) \notin \xi$ and $(t_2, t_1) \notin \xi$.*

5. Auxiliary results (c)

In this section, let $z_i \in Z$, $r_i, s_i \in A^*$ and $t_i = r_i z_i s_i$, $i = 1, 2$, be such that $(t_1, t_2) \notin \xi$ and $(t_2, t_1) \notin \xi$ (see the preceding section). Put $(P(t_1, t_2) =)\ P = \{w \in A^* \,|\, (w, t_1) \in \xi, (w, t_2) \in \xi\}$ and denote by $(Q(t_1, t_2) =)\ Q$ the set of $w \in P$ such that $w = w'$ whenever $w' \in P$ and $(w, w') \in \xi$.

Lemma 5.1.
(i) *If $w \in P$, then $(w, t_1) \in \tau$, $(w, t_2) \in \tau$ and $t_1 \ne w \ne t_2$.*
(ii) *If $w \in P$, then $w \in Q$ if and only if $(w, w') \notin \rho$ for every $w' \in P$.*

Proof. Easy. □

Remark 5.2. Assume that $P \ne \emptyset$. By III.6.4 (i), there exists at least one word $t \in A^*$ with $(t_1, t) \in \xi$ and $(t_2, t) \in \xi$. Then $(t_1, t) \in \tau$ and $(t_2, t) \in \tau$. Furthermore, if r_i, s_i are reduced, then $(r_1\psi(z_1)s_1, t) \in \xi$ and $(r_2\psi(z_2)s_2, t) \in \xi$.

Lemma 5.3. *Assume that the relation ρ is regular (e.g., if ψ is strictly length decreasing – see III.7.7). Then for every $w \in P$ there exists at least one $w' \in Q$ with $(w, w') \in \xi$.*

Proof. Put $R = \{v \in P \,|\, (w, v) \in \xi\}$ and $M = \{\mathrm{dist}(v, t_1) + \mathrm{dist}(v, t_2) \,|\, v \in R\}$. Then M is a non-empty set of positive integers and if $w' \in R$ is such that $\mathrm{dist}(w', t_1) + \mathrm{dist}(w', t_2)$ is the smallest number in M, then $w' \in Q$. Notice that if ψ is strictly length decreasing and $w' \in R$ is such that $|w'|$ is the smallest number in $|R|$, then $w' \in Q$. □

Now, take $w \in Q$ and let $w_0^{(i)}, w_1^{(i)}, \dots, w_{m_i}^{(i)}$, $m_i \ge 1$, $i = 1, 2$, be ρ-sequences such that $w_0^{(i)} = w$ and $w_{m_i}^{(i)} = t_i$.

Lemma 5.4. *If $(w_j^{(i_1)}, w_k^{(i_2)}) \in \xi$ for $\{i_1, i_2\} = \{1, 2\}$ and some $0 \le j \le m_{i_1}$, $0 \le k \le m_{i_2}$, then $w_j^{(i_1)} = w$.*

Proof. We have $(w_j^{(i_1)}, t_{i_1}) \in \xi$ and $(w_k^{(i_2)}, t_{i_2}) \in \xi$. Since $i_1 \ne i_2$, it follows that $w_j^{(i_1)} \in P$. But $w \in Q$ and $(w, w_j^{(i_1)}) \in \xi$. Consequently, $w_j^{(i_1)} = w$. □

Lemma 5.5. $w_1^{(1)} \neq w_1^{(2)}$.

Proof. If $w_1^{(1)} = v = w_1^{(2)}$, then $v \in P$, $(w,v) \in \rho$ and $v = w$, since $w \in Q$. Thus $(w,w) \in \rho$, a contradiction with the irreflexivity of ρ. □

Lemma 5.6. *Assume that either τ is irreflexive (see III.3.3) or that the sum $m_1 + m_2$ is minimal (for the word w). Then:*

(i) $(w_j^{(i_1)}, w_k^{(i_2)}) \notin \xi$ *for all* $\{i_1, i_2\} = \{1, 2\}$, $0 \le j \le m_{i_1}$, $0 \le k \le m_{i_2}$.

(ii) *All the words* $w = w_0^{(1)} = w_0^{(2)}$, $w_{j_i}^{(i)}$, $j_i = 1, 2, \dots, m_i$, $i = 1, 2$, *are pair-wise different.*

Proof. Easy (use 5.4). □

Lemma 5.7. $\mathrm{tr}(w) \ge 2$ *(i. e., w is not meagre).*

Proof. Clearly, w is not reduced. On the other hand, if $\mathrm{tr}(w) = 1$, then $w = pzq$, where $z \in Z$ and p, q are reduced. Consequently, $w_1^{(i)} = p\psi(z)q$, $w_1^{(1)} = w_1^{(2)}$, a contradiction with 5.5. □

Lemma 5.8. $\mathrm{alph}(w) \subseteq \mathrm{alph}(t_1) \cup \mathrm{alph}(t_2)$.

Proof. Let, on the contrary, $w = pzq$, where $z \in Z$ and $z \notin \mathrm{alph}(t_1) \cup \mathrm{alph}(t_2)$. Using 2.3 and 2.4, we get ρ-sequences $v_0^{(i)}, v_1^{(i)}, \dots, v_{m_i}^{(i)}$, $i = 1, 2$, such that $v_0^{(i)} = w$, $v_1^{(i)} = p\psi(z)q$ and $v_{m_i}^{(i)} = t_i$. Then $v_1^{(i)} = v$, where $(v, t_i) \in \xi$, $v \in P$ and $v = w$ a contradiction with the irreflexivity of ρ. □

Lemma 5.9. *Assume that $z_1 \neq z_2$ and $z_1 \notin \mathrm{alph}(r_2) \cup \mathrm{alph}(s_2)$ (i. e., $z_1 \notin \mathrm{alph}(t_2)$). If $w = p_0 z_1 q_0$, then the sequence $w = w_0^{(1)}, w_1^{(1)}, \dots, w_{m_1}^{(1)} = t_1$ is $(z_1, \alpha_0, \dots, \alpha_{m_1})$-fluent, where $\alpha_0 = (p_0, z_1, q_0)$, $\alpha_1 = (p_1, z_1, q_1)$, $\alpha_{m_1} = (p_{m_1}, z_1, q_{m_1})$, $p_{m_1} = r_1$, $q_{m_1} = s_1$ (then $(p_0, r_1) \in \xi$ and $(q_0, s_1) \in \xi$).*

Proof. Proceeding by contradiction, assume that our result is not true. According to 2.3 and 2.4, there is a ρ-sequence $w = v_0^{(2)}, p_0\psi(z_1)q_0 = v_1^{(2)}, v_2^{(2)} \dots, v_{m_2}^{(2)} = t_2$. Thus $v_1^{(1)} = v = v_1^{(2)}$, $(v, t_1) \in \xi$, $(v, t_2) \in \xi$, $v \in P$ and $v = w$, $(w, w) \in \rho$, a contradiction with the irreflexivity of ρ. □

Lemma 5.10. *Assume that $z_1 \neq z_2$ and $z_1 \notin \mathrm{alph}(r_1) \cup \mathrm{alph}(r_2) \cup \mathrm{alph}(s_2)$. Then $w \neq y_0 z_1 y_1 z_1 y_2$ for all $y_0, y_1, y_2 \in A^*$.*

Proof. Let, on the contrary, $w = y_0 z_1 y_1 z_1 y_2$. Then $(y_0 z_1 y_1, r_1) \in \xi$ and $(y_2, s_1) \in \xi$ by 5.9 and 2.1 (ii). Since $z_1 \notin \mathrm{alph}(r_1)$, we have $(y_0\psi(z_1)y_1, r_1) \in \xi$ by 2.4, and therefore $(y_0\psi(z_1)y_1 z_1 y_2, t_1) \in \xi$. On the other hand, $z_1 \notin \mathrm{alph}(t_2)$, and so $(y_0\psi(z_1)y_1 z_1 y_2, t_2) \in \xi$ as well. Thus $y_0\psi(z_1)y_1 z_1 y_2 \in P$, a contradiction with $w \in Q$ and $\psi(z_1) \neq z_1$. □

Proposition 5.11. *Assume that* $z_1 \neq z_2$ *and* r_i, s_i *are reduced,* $i = 1, 2$. *Then there exist reduced words* $x_0, x_1, x_2 \in A^*$ *such that just one of the following two cases takes place:*

(1) $w = x_0z_1x_1z_2x_2$, $x_0 = r_1$, $(x_1z_2x_2, s_1) \in \tau$, $(x_0z_1x_1, r_2) \in \tau$ *and* $x_2 = s_2$ *(then* $w = r_1z_1x_1z_2s_2$, $(x_1\psi(z_2)s_2, s_1) \in \xi$, $(r_1\psi(z_1)x_1, r_2) \in \xi$ *and* r_1, s_2 *are reduced);*
(2) $w = x_0z_2x_1z_1x_2$, $x_0 = r_2$, $(x_1z_1x_2, s_2) \in \tau$, $(x_0z_2x_1, r_1) \in \tau$ *and* $x_2 = s_1$ *(then* $w = r_2z_2x_1z_1s_1$, $(x_1\psi(z_1)s_1, s_2) \in \xi$, $(r_2\psi(z_2)x_1, r_1) \in \xi$ *and* r_2, s_1 *are reduced).*

Proof. Combining 5.7, 5.8 and 5.10 (and the dual), we see that $\mathrm{tr}(w) = 2$ and $\mathrm{alph}(w) = \{z_1, z_2\}$. According to I.6.4, either $w = x_0z_1x_1z_2x_2$ or $w = x_0z_2x_1z_1x_2$, where x_0, x_1 and x_2 are reduced. Assume the former equality, the latter being dual. Now, it follows from 5.9 that $(x_0, r_1) \in \xi$. Since x_0 is reduced, we get $x_0 = r_1$. Furthermore, $(x_1z_2x_2, s_1) \in \xi$ and, since $z_2 \notin \mathrm{alph}(s_1)$, we have $(x_1z_2x_2, s_1) \in \tau$. The rest is similar. □

Remark 5.12. Consider the situation from 5.11 (and its proof) and assume that (1) is true (the other case being dual). Put $u_1 = x_1\psi(z_2)s_2$ and $u_2 = r_1\psi(z_1)x_1$. We have $(u_1, s_1) \in \xi$ and $(u_2, r_2) \in \xi$.

(i) If u_1 is reduced, then $u_1 = s_1$, $t_1 = r_1z_1x_1\psi(z_2)s_2$, $(w, t_1) \in \rho$ and $(t_1, u_3) \in \rho$, where $u_3 = r_1\psi(z_1)x_1\psi(z_2)s_2$.
(ii) If u_2 is reduced, then $u_2 = r_2$, $t_2 = r_1\psi(z_1)x_1z_2s_2$, $(w, t_2) \in \rho$ and $(t_2, u_3) \in \rho$, where $u_3 = r_1\psi(z_1)x_1\psi(z_2)s_2$.
(iii) If the equivalent conditions of II.7.3 are satisfied, then all the words u_1, u_2, s_1, r_2 are meagre. Now, if s_1 is not reduced, then $s_1 = y_0z_3y_1$, $z_3 \in Z$, $z_1 \neq z_3 \neq z_2$, y_0, y_1 are reduced and $t_1 = r_1z_1y_0z_3y_1$. If r_2 is not reduced, then $r_2 = y_2z_4y_3$, $z_4 \in Z$, $z_1 \neq z_4 \neq z_2$, y_2, y_3 are reduced and $t_2 = y_2z_4y_3z_2s_2$.

Remark 5.13. Assume that $P \neq \emptyset$ and choose $w' \in P$ such that $m'_1 + m'_2$ is minimal, where m'_1 and m'_2 is the length of a ρ-sequence from w' to t_1 and t_2, resp. It is easy to see that 5.5, 5.7, 5.8 and 5.9 remain true.

6. The ultimate consequence

Theorem 6.1. *Assume that the mapping* ψ *is strictly length decreasing. Let* $z_1, z_2 \in Z$ *and* $r_1, r_2, s_1, s_2 \in A^*$ *be such that* $z_1 \neq z_2$, *the words* r_1, r_2, s_1, s_2 *are reduced and* $P(t_1, t_2) \neq \emptyset$, *where* $t_1 = r_1z_1s_1$ *and* $t_2 = r_2z_2s_2$. *Then* $Q(t_1, t_2) \neq \emptyset$ *and, if* $w \in Q(t_1, t_2)$, *then just one of the following two cases takes place:*

(1) $w = r_1z_1xz_2s_2$, $(r_1z_1x, r_2) \in \tau$, $(xz_2s_2, s_1) \in \tau$ *and* x *is reduced;*
(2) $w = r_2z_2yz_1s_1$, $(r_2z_2y, r_1) \in \tau$, $(yz_1s_1, s_2) \in \tau$ *and* y *is reduced.*

Proof. By 5.3, $Q(t_1, t_2) \neq \emptyset$. The rest follows from 5.11. □

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