Vanishing of Tor over complete intersections

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Let $(R, \mathfrak{m})$ be a Noetherian local ring, and let M and N be non-zero finitely generated R-modules. One says that M and N are *Tor-independent* provided $\operatorname{Tor}_i^R(M, N) = 0$ for every $i > 0$. In this talk we will seek conditions on M, N, and $M \otimes_R N$ that force M and N to be Tor-independent. One reason for seeking such conditions is that there are many situation in which Tor-independence implies the *depth formula*:

$$\operatorname{depth} M + \operatorname{depth} N = \operatorname{depth}(M \otimes_R N) + \operatorname{depth} R$$

Auslander proved, more than 50 years ago, that Tor-independence implies the depth formula if one of the modules has finite projective dimension. About 20 years ago, Huneke and I proved that Tor-independence implies the depth formula if R is a complete intersection (a local ring of the form $S/(\underline{f})$, where $(\underline{f}) = (f_1, \dots, f_c)$ is a regular sequence). (There are no known examples where Tor-independence does *not* imply the depth formula.)

The talk will be guided by the following

Conjecture. *Suppose that $R = S/(\underline{f})$ as above and, in addition, that R is a domain. If $M \otimes_R N$ satisfies Serre's condition (S_{c+1}), then M and N are Tor-independent (and hence the depth formula holds).*

The case $c = 0$ (that is, R is a regular local ring) was proved by Auslander and Lichtenbaum in the sixties. The case $c = 1$ was proved by Huneke and me in our 1994 paper. I will discuss some new tools for attacking this problems and give some positive results.

This research is joint work with Olgur Celikbas, Srikanth Iyengar, and Greg Piepmeyer.